Coexisting with Zombies: Finding an Equilibrium During a Zombie Apocalypse

Marvyn Bailly

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Abstract

This paper combines the pop culture zombie with the renowned SIR disease model. I introduce new assumptions on zombies and modified the existing SIR zombie models to create more 'realistic' equations. I continue to add a new category to the model that differentiates the existing removed category. The differential equations are then analyzed using Jacobian matrices to find the stability of equilibrium points. Finally, I present solutions that may be used to regulate the zombie virus to find an equilibrium where humans and zombies may coexist without the extension of other species.

1 Introduction

A famous pop culture event is the zombie apocalypse, where a vicious disease spreads through the world, infecting millions. The virus reincarnates the dead, making them highly aggressive towards the living and spreading the disease. In many fictional zombie tales, the virus destroys humanity, bringing the earth to an apocalyptic stage where the final humans fight against the zombies. The popular proposed solution to the apocalypse is to completely exterminate the zombies or fend against them long enough for a cure to be found.

This research paper will analyze the zombie virus using mathematical models based on data drawn from highly acclaimed films, other academic research, and personal ideas. I will seek to answer if the zombie virus can be handled in a more ethical manner. The solutions to the proposed questions will give us insight and knowledge in the event of a zombie outbreak and the mathematical process can be applied to other large-scale epidemic and pandemics. With a better understanding of the subject, humanity can prepare for a doomsday situation, equipping us with the necessary and information tools to coexist with zombies rather than going to war with them.

2 Background

While mathematicians have been modeling virus spread for a long time, only in the 20th century were popular and effective disease models applied to zombies. A particular interest was placed in Munz et al. [1]; they modeled the zombie outbreak using a basic SIR virus model and built upon it to include various new ideas and complexity fitting for zombies. The basic SIR model can be considered a flow chart with three categories, Susceptible, Infectious, and Recovered (SIR). The susceptible population is at risk of being exposed to the virus. Once individuals have contracted the disease, they are moved into the infectious category, spreading the disease. Once they have recovered, they are moved into the final category.

Munz et al. [1] modified this concept into a model with four categories, Susceptible, Infected, Zombie, and Removed (SIZR), which created a more realistic representation of a zombie outbreak (see Figure 1). Like the SIR model, the susceptible population is at risk of becoming infected or dying of natural causes (natural deaths are placed in the removed category). Once an induvial is infected, they move to the infected category for a set period before transitioning into a zombie. During the infected phase, the individual still has a chance of dying naturally. Once a human has become a zombie, they spread the disease but may be killed and placed into the removed category. The final aspect of the SIZR model is that all members of the removed population can be reincarnated as a zombie.

The SIZR model is built upon by Witkowski and Blais [2], who use data from pop culture films to analyze different types of zombies and scenarios. Both pieces of work lean towards eradicating zombies through organized and fast attacks while working on a cure as the differential equations used to describe the SIZR model tend to be unstable. Witkowski and Blais continue to explain that despite the comical and fictional nature of the zombie, the equations and trends can be used to combat other diseases such as influenza.

2.1 Difference and Expectations

I plan on combining the two works by modifying the SIZR model to fit new conditions and categories created from data drawn from movies and video games. I will add a new category for potential (P) zombies (see Figure 1) and thus call my proposed model SIZRP. I propose that the only way a human can kill a zombie is to behead them or burn them, placing them into the removed category. In a natural death where the human's head is preserved, they will be placed into the potential category where the body has a fixed chance of becoming a zombie; I will call this process zombification. I also propose that the zombie body naturally decomposes over time, meaning a fixed natural death rate for the zombies. I hope that my new assumptions on the zombie model will stabilize the differential equations, allowing for interesting solutions to zombie outbreaks. For example, humans coexisting with zombies or regulating the number of zombies through mass killings. I hope this new outlook on the zombie apocalypse can be built to create a new perspective on zombies that do not involve the extinction of zombies. Furthermore, this method of analysis can be applied to current and future diseases to find new solutions.

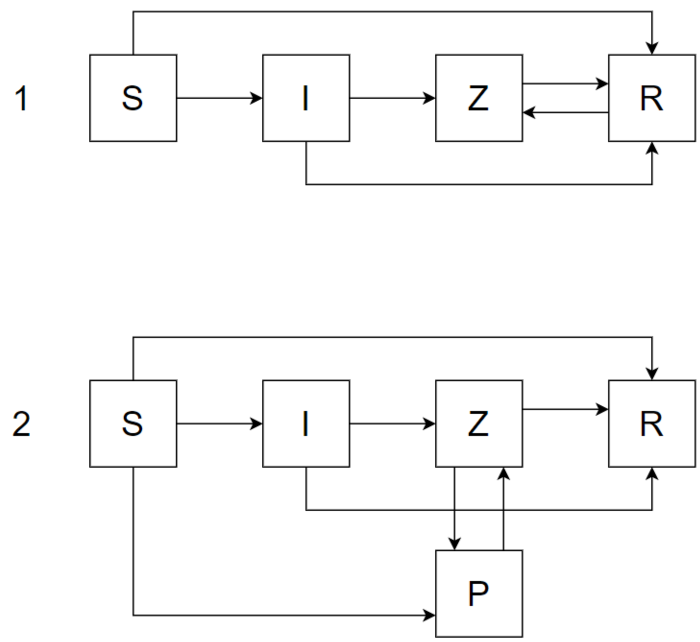

Figure 1. (1) Munz et al. [1] SIZR model. (2) SIZRP (proposed model)

3 Model

3.1 Assumptions and Parameters

The SIZRP model consists of five categories:

- Susceptible (S)
- Infected (I)
- Potential (P)
- Zombie (Z)
- Removed (R)

Although some of the assumptions are like [1] and [2], introducing the new category P introduces more parameters and assumptions. To begin, all alive humans begin in the susceptible category at the start of the zombie infection. People who die from non-zombie-related causes, such as old age, are moved into the potential category (parameter γ) under the following criteria. Their head is still attached to their body, and their body is still in moving condition. If the criteria are not met, they are placed into the removed category (parameter δ). The difference between the potential and removed categories is that potentials can be zombified (parameter ζ) while the latter is "safe" from the infection.

When a potential human can only be infected when bitten by a zombie, I will assume that the primary purpose of zombies is to spread the virus, as seen in *World War Z* [3], meaning that they will only bite humans rather than killing them. Additionally, zombies only feed upon humans, removing the case of infected animals. This means that when potential human faces a zombie, three events may occur:

- The zombie successfully infects the human, placing the human into the infected category, by biting them (parameter β)
- The human temporarily defeats the zombie by cutting off a limb, placing the zombie in the potential category (parameter α)
- The human permanently defeats the zombie by either cutting off their head or by burning their body, placing the zombie in the removed category (parameter ν)

A human remains infected for a short period before the virus either kills them or dies to other non-zombie-related causes due to their weakened immune system (parameter σ), placing them into the removed. If the human survives the infection time, they are zombified (time period parameter ρ). Since the model is searching for equilibrium, it will analyze a long period of time. Thus, there will be a constant birth rate, Π, and a constant death rate of zombies, χ. The death of a zombie will occur once the dead body decomposes to a point where the virus may no longer use it.

3.2 Mathematical Equations

The following dynamic equations can model the assumptions and observations:

$$S' = \frac{dS}{dt} = \Pi - \beta SZ - \delta S - \gamma S \tag{1}$$

$$I' = \frac{dI}{dt} = \beta SZ - \rho I - \sigma I \tag{2}$$

$$Z' = \frac{dZ}{dt} = \zeta P + \rho I - \alpha SZ - \nu SZ - \chi Z \tag{3}$$

$$R' = \frac{dR}{dt} = \delta S + \sigma I + \nu SZ + \chi Z \tag{4}$$

$$P' = \frac{dP}{dt} = \gamma S + \alpha SZ - \zeta P \tag{5}$$

And is illustrated in figure 2.

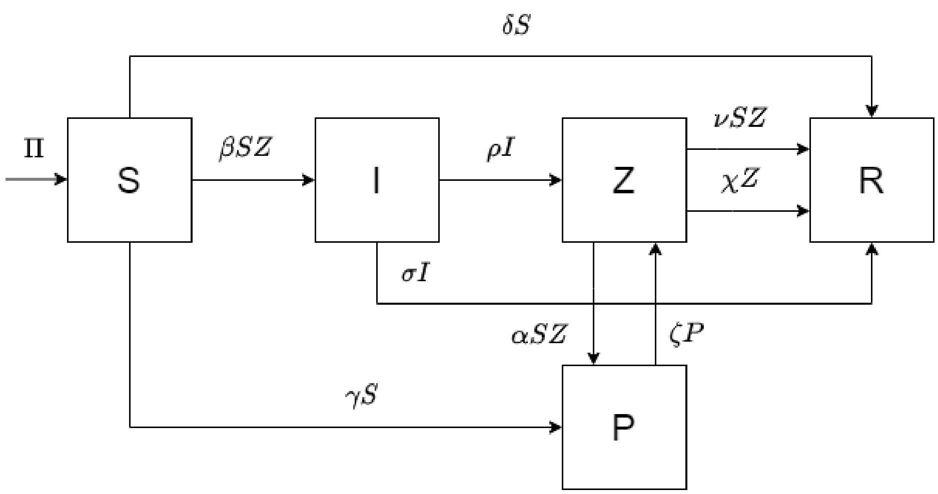

Figure 2. SIZRP model with parameters.

4 Result

The proposed SIZRP equations form a nonlinear system of ordinary differential equations that can be analyzed to find equilibrium solutions. The equations are different to [1] and [2] as introducing a new category affects all of the equations that hopefully allow for human-zombie coexistence, unlike in [1] and [2]. Once the points have been found, I will know if the zombie infection will naturally die out or if humanity will face a doomsday scenario. Depending on the result, I will introduce different ideas, such as yearly zombie killings, to find a situation, under reasonable parameters, where zombies and humans may coexist on earth.

For the sake of simplicity, I will assume that $\Pi = 0$. To find an equilibrium solution to the SIZRP model, let us set the differential equations equal to zero,

$$\beta SZ - \delta S - \gamma S = 0 \tag{6}$$
$$\beta SZ - \rho I - \sigma I = 0 \tag{7}$$
$$\zeta P - +\rho I - \alpha SZ - \nu SZ - \chi Z = 0 \tag{8}$$
$$\delta S + \sigma I + \nu SZ + \chi Z = 0 \tag{9}$$
$$\gamma S + \alpha SZ - \zeta P = 0 \tag{10}$$

For the first equality to hold, $\mathrm{S} = 0$ or $Z = \frac{\delta+\gamma}{\beta}$. Meaning that one equilibrium point results in a doomsday scenario where all humans die ($\mathrm{S} = 0$)

$$(\bar{S}, \bar{I}, \bar{Z}, \bar{R}, \bar{P}) = (0,0,\bar{Z},0,0) \tag{11}$$

or when the zombie population reaches a set point.

$$(\bar{S}, \bar{I}, \bar{Z}, \bar{R}, \bar{P}) = (\mathrm{A},0,0,0,0) \text{ where A is some constant.} \tag{12}$$

This setpoint is dependent on the infection rate and non-zombie-related deaths that move the susceptible to the removed and potential category. These results intuitively make sense. Next, I will calculate the Jacobian matrix and plug in the points.

The Jacobian is

$$J = \begin{bmatrix} -\beta Z - \delta - \gamma & 0 & \beta S & 0 & 0 \\ \beta Z & \rho - \sigma & 0 & 0 & 0 \\ -\alpha Z - \nu Z & \rho & -\alpha S - \nu S - \chi & 0 & \zeta \\ \delta + \nu Z & \sigma & \nu S + \chi & 0 & 0 \\ \gamma + \alpha Z & 0 & \alpha S & 0 & -\zeta \end{bmatrix}. \tag{13}$$

Therefore, the Jacobian at $\mathrm{S} = 0$

$$J\left(0,0,\bar{Z},0,0\right)=\begin{bmatrix} -\beta\bar{Z}-\delta-\gamma & 0 & 0 & 0 & 0 \\ \beta\bar{Z} & \rho-\sigma & 0 & 0 & 0 \\ -\alpha\bar{Z}-\nu\bar{Z} & \rho & -\chi & 0 & \zeta \\ \delta+\nu\bar{Z} & \sigma & \chi & 0 & 0 \\ \gamma+\alpha\bar{Z} & 0 & 0 & 0 & -\zeta \end{bmatrix}. \quad (14)$$

The Jacobian has an eigenvalue with a root with a real positive component, making the solution unstable, which means a disease-free situation is impossible.

The Jacobian at the other equilibrium solution, $Z=\frac{\delta+\gamma}{\beta}$ is

$$J(\mathrm{A},0,0,0,0) \ = \begin{bmatrix} 0 & 0 & \beta\mathrm{A} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha\mathrm{A}-\nu\mathrm{A} & 0 & \zeta \\ 0 & 0 & \nu\mathrm{A}+\chi & 0 & 0 \\ \gamma & 0 & \alpha\mathrm{A} & 0 & -\zeta \end{bmatrix}. \quad (15)$$

The Jacobian has an eigenvalue with a root with a real positive component, meaning that this equilibrium solution is also unstable. Thus, there is no situation where zombies reach a stable equilibrium. Furthermore, this means that the SIZRP equations can not support a natural human-zombie equilibrium.

5 Conclusion

5.1 Further Work

While no equilibrium supports human and zombie coexistence, humans can take actions to try and 'balance' the equations. One possible solution could be to organize 'sacrifices' (parameter τ) to either reduce the spread of the virus, killing zombies, or help the virus grow, killing humans. In the case that $Z\to\infty$ as $t\to\infty$, the SIZRP equations can be modified as follows,

$$Z'=\frac{dZ}{dt}=\zeta P-\alpha SZ-\nu SZ-\chi Z-\tau Z \quad (16)$$

$$R'=\frac{dR}{dt}=\delta S+\sigma I+\nu SZ+\chi Z\ +\ \tau Z. \quad (17)$$

This might help reduce the fast spread of the disease. Every time period, a highly trained group of humans could go out and a number of zombies that is proportional to the zombies' population. The reduction may help to regulate the infection.

On the other hand, in the case that $S\to\infty$ as $t\to\infty$,

$$S'=\frac{dS}{dt}=\Pi-\beta SZ-\delta S-\gamma S\ -\tau S \quad (18)$$

$$P'=\frac{dP}{dt}=\gamma S+\alpha SZ-\zeta P\ +\ \tau S. \quad (19)$$

When the zombie population is suffering, a number of humans that is proportional to the human population would be killed in a way that places them into the potential category. This way, the killings of the humans do not depend on the zombie population and will allow their bodies to be resurrected.

However, the above solution is still rather barbaric and does not seem better than having one population kill the other. What if there was a situation where zombies did not attack humans? A new model could be made where the I category is removed, an SZRP model. This would give the following equations,

$$S' = \frac{dS}{dt} = \Pi - \delta S - \gamma S \qquad (20)$$

$$Z' = \frac{dZ}{dt} = \zeta P - \alpha SZ - \nu SZ - \chi Z \qquad (21)$$

$$R' = \frac{dR}{dt} = \delta S + \nu SZ + \chi Z \qquad (22)$$

$$P' = \frac{dP}{dt} = \gamma S + \alpha SZ - \zeta P \qquad (23).$$

A similar process can be used to analyze the stability of the equations to see if they support a stable human-zombie equilibrium. Although this scenario seems unlikely, it is not unheard of. African villages have been able to share land and finite resources with lions. This idea has also been played in [4], where a human woman falls in love with a zombie. This outcome is the optimal reaction to the zombie outbreak as it allows for peace between the species. Furthermore, it would be interesting to use data drawn from pop culture pieces that support coexistence, such as [4], and test the validity of the proposed model.

5.2 Final Thoughts

While the SIZRP model I created did not support the solutions I am searching for, the paper demonstrates the process of adding new categories and studying their behavior. This is a powerful tool for finding a solution to a problem that can be modeled with the basic SIR model. In my case, I began with the basic SIR model and tried to find a model that would allow for a more ethical way to handle the zombie virus outbreak. If a model supports this outcome, it can be used as a basis for how humans react. This paper highlights how the basic SIR model can be modified to fit many situations, allowing for more diverse use while studying diseases and similar situations. A quarantine stage can be applied to make the model more applicable to real world virus. The parameters of the equations can be modified to show the importance of washing hands, wearing masks, and other protective means. While the zombie apocalypse may be purely fictional, the presented model and procedure have many important nonfictional applications.

Reference

[1] Munz, et al. "When Zombies Attack!: Mathematical Modeling of an Outbreak of Zombie Infection,", edited by Tchuenche and Chiyaka, *Nova Science Publishers, Inc,* 2009 , pp. 133-150.

[2] Blais and Witkowski. "Bayesian Analysis of Epidemics - Zombies, Influenza, and other Diseases" Science and Technology Department.

[3] Forster, Marc, (director) 2013 *World War Z* (film)

[4] Levine, Jonathan, (director and writer) 2013 *Soft Bodies*